Related Rates

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How to solve related rates problems

(1) **Draw a picture!**, labeling a couple of variables. **HOWEVER** do not put any numbers on your picture, except for constants! Otherwise you'll get confused later on

(2) Figure out what you *ultimately* want to calculate, and don't lose track of it

(3) Find an equation relating your variables

(4) Differentiate your equation using the chain rule/implicit differentiation.

(5) **NOW** plug in all the numbers you know! Sometimes, you might need to calculate a number of 'missing variables'. Here an extra picture as in 1), but with all the numbers plugged in, might be useful

(6) Solve for whatever you were looking for in 2)

List of tricks

– Pythagorean theorem, Law of similar triangles

– Definition of sin and cos; Law of sines and cosines

– Formulas for areas and/or volumes:

- Volume of a cone: $V = \frac{\pi}{3} r^2 h$
- Volume of a cylinder: $V = \pi r^2 h$
- Volume of a ball: $V = \frac{4}{3}\pi r^3$, Surface area of a sphere: $S = 4\pi r^2$

Problem 1

The altitude of a triangle is increasing at a rate of 1 cm/min while the area of the triangle is increasing at a rate of 2 cm^2/min. At what rate is the base of the triangle changing when the altitude is 10 cm and the area is $100cm^2$?

Problem 2

Two cars start at the same point. Car A travels North at a rate of 6 mi/h and Car B travels East at a rate of 2.5 mi/h. At what rate is the distance between the two cars increasing 2 hours later?

Problem 3

A ladder 10 feet long rests against a vertical wall. The bottom of the ladder slides away from the wall at a rate of 1 ft/s. How fast is the angle between the ladder and the wall changing when the bottom is 6 feet from the wall?

Problem 4

A particle is moving along the curve $xy + x^2y^2 = 6$ in the $xy-$plane. At the moment it passes through the point $(2, 1)$, its $x-$ coordinate is decreasing at 2 units/sec. How fast is its $y-$coordinate changing at this moment?

Problem 5

Suppose that the surface area of a sphere is increasing at a rate of 5 cm^2 per second. How fast it the volume of that ball increasing when its radius is 2 cm?

Problem 6

Assume Peyam's utility function is given by $U = G^2\sqrt{C}$, where L is the number of utils (happiness points) due to GSI-ing for Math 1A lectures, and C is the number of utils due to eating cake. If currently $G = 10$ and is increasing by 4 utils/day and $C = 100$ and is decreasing by 10 utils/day, is Peyam getting happier or sadder now, and at what rate?

Problem 7

(HARD) Suppose that the minute hand of a certain clock is 15 mm long and the hour hand is 12 mm. How fast is the distance between the hour hand and the minute hand of the clock changing at 2 pm?